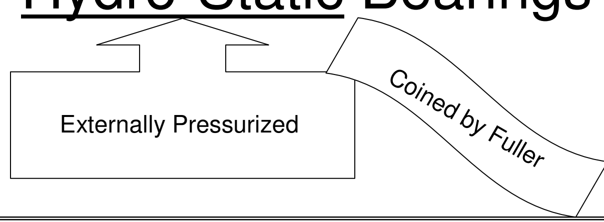
Hydrostatic Bearings: An Introduction

Dr. H. Hirani
Dept. of Mech. Eng.
I.I.T. Bombay
hirani@me.iitb.ac.in

Design Procedure for <u>Hydro-Static</u> Bearings (HSB)



- Invented by <u>L. D. Girard</u> (Frenchman).
- Completely removal of wear and reduction of coefficient of friction to 1/500.
- Machines using hydrostatic support show a <u>better rotational accuracy</u> within 2 micro-inch (0.051 micron) and RMS surface roughness down to 0.25 micro-inch (0.00635 micron)

Features of Hydrostatic support

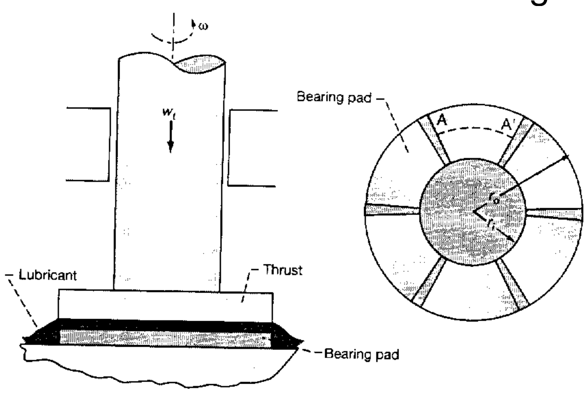
- Surfaces can be separated by full fluid film even at zero speed.
  - No problem with micro roughness and waviness.
- Zero friction at zero speed.
  - Useful feature for large size telescopes and radars.
- High stiffness
  - Oil film thickness varies as cube root of load. $h \propto W^{1/3}$
- Why not every bearing is based on Hydrostatic mechanism
  - High pressure supply… Reliability & life of high pressure oil lines are always in doubt.

HSB for Large Binocular Telescope

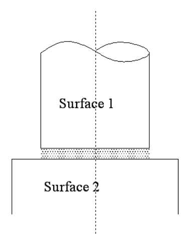
- Supports for Azimuth and Elevation axes.
  - Telescope makes a far away object look closer by collecting light from a distant object (objective lens or primary mirror) and brings that light (image) to a focus where a second device (eyepiece lens) magnifies the image and brings it to our eye.
  - A telescope's ability to collect light is directly related to the diameter of the lens or mirror -- the **aperture** -- that is used to gather light. Generally, the larger the aperture, the more light the telescope collects and brings to focus, and the brighter the final image.
  - Refractors have good resolution, high enough to see details. However, it is difficult to make large objective lenses (greater than 4 inches or 10 centimeters) for refractors. Because the aperture is limited, a refractor is less useful for observing faint, deep-sky objects, like galaxies and nebulae, than reflector types of telescopes.

Thrust Bearings

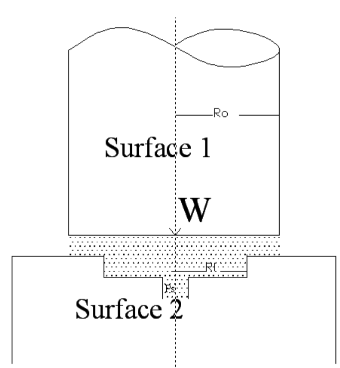
- Many loads carried by rotating machinery have components that act in the direction of the shaft's axis of rotation. Bearings supporting such loads are known as thrust bearings.



Elementary 1-D Analysis



- Assume a shaft of radius R_o is located co-axially with a circular recess of radius R_i.
- Assume all the oil in recess is at the supply pressure P_s.



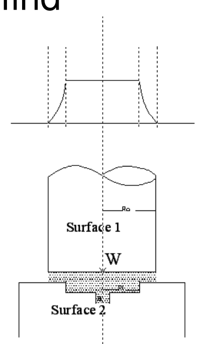
- Consider a small element of angular extent dθ at a radius r and radial width dr.
- Elemental flow rate: $\delta q = -\frac{h^3}{12\eta}\frac{dp}{dr}.rd\theta$
- If flow is symmetrical to the origin, and radial flow rate is constant, then flow rate:

$$Q = -\frac{h^3}{12\eta}\frac{dp}{dr}.r.2\pi$$

- If film thickness is constant, then on integration:

$$\frac{\pi h^3 p}{6\eta} = -Q\,(\log r + C_1)$$

- Using two boundary conditions to find unknown values of C_1 and Q



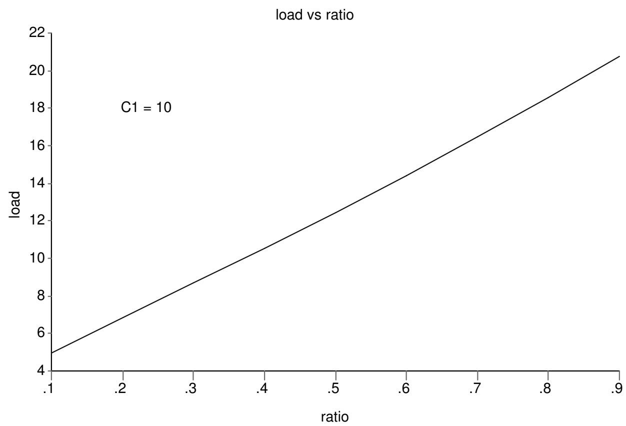
$$p = p_s \frac{\log {R_0}/{r}}{\log {R_0}/{R_i}} \quad \text{in the region } R_0 \geq r \geq R_i$$

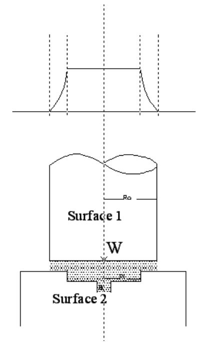
- Load carrying capacity:

$$W = p_s.\pi R_i^2 + \int_{R_i}^{R_o}\int_0^{2\pi} p(rd\theta)dr$$

- Substituting expression of *p* and rearranging

$$W = p_s.(\pi R_o^2)\left(\frac{1-{R_i^2}/{R_o^2}}{2.\log\left({R_o}/{R_i}\right)}\right) \longrightarrow W = C_1\left(\frac{1-r_1^2}{2.\log\left({1}/{r_1}\right)}\right)$$





$$W = p_s . \pi R_i^2 + \int_{R_i}^{R_o} \int_0^{2\pi} p(r d\theta) dr$$

$$Q = \frac{\pi h_0^3 p_s}{6\eta} \frac{1}{\log(1/r_1)} \longrightarrow Q = C_2 \frac{1}{\log(1/r_1)}$$

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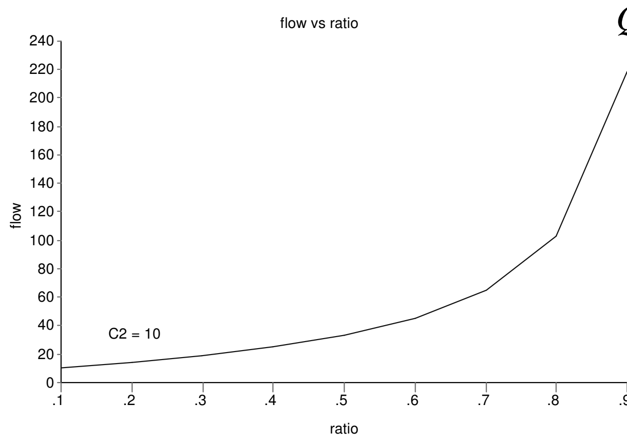


- Load is not a function of film thickness (h_0), but flow is a very strong function of film thickness.
- Film thickness is designed based on surface finish & vibration.

Power loss

- Power consumption in the hydrostatic bearing system consists of pumping power and friction losses.

$$P_t = P_h + P_f$$

$$P_h = Q.P_s$$

$$P_f = \frac{\pi}{2}\eta\frac{R_0^4}{h_0}\left(1-\left(\frac{R_i}{R_0}\right)^4\right)\omega^2$$

$$F = \eta A\frac{U}{h_0}$$ ← Petroff equation

$$F = \eta\frac{\omega r}{h_0}A(r)$$

$$P_f = F\omega r \Rightarrow P_f = \eta\frac{\omega^2}{h_0}\int_{R_i}^{R_0} 2\pi r^3 dr$$

$$P_t = \frac{1}{\eta_1}\frac{1}{6\eta}\frac{\pi h_0^3}{\log(R_0/R_i)}P_s^2 + \frac{1}{\eta_2}\frac{\pi}{2}\eta\frac{R_0^4}{h_0}\left(1-\left(\frac{R_i}{R_0}\right)^4\right)\omega^2$$

<u>Example</u>: W = 1000 N, ω=5000 rpm, R_0=100 mm, R_i=50 mm, η=0.01 Pa.s, η_1=0.6, η_2=0.9. Optimize minimum film thickness for minimum power loss

$$P_t = \frac{1}{\eta_1}\frac{1}{6\eta}\frac{\pi h_0^3}{\log(R_0/R_i)}P_s^2 + \frac{1}{\eta_2}\frac{\pi}{2}\eta\frac{R_0^4}{h_0}\left(1-\left(\frac{R_i}{R_0}\right)^4\right)\omega^2$$

$$W = p_s.\left(\pi R_o^2\right)\left(\frac{1-R_i^2/R_o^2}{2.\log\left(R_o/R_i\right)}\right)$$

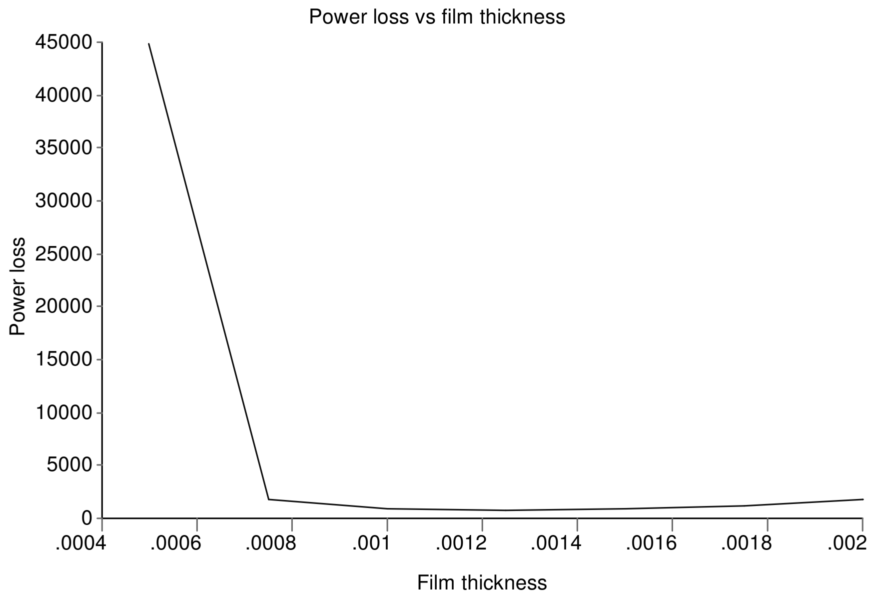
$$P_t = C_1 h_0^3 + C_2/h_0$$

$$\omega = 2\pi * 5000/60 \Rightarrow \omega = 523.6 \text{ rad/s}$$

$$P_s = \frac{1000}{\pi * 0.1^2}\frac{2.\log(2)}{\left(1-0.5^2\right)} \Rightarrow P_s = 58{,}824 \text{ Pa}$$

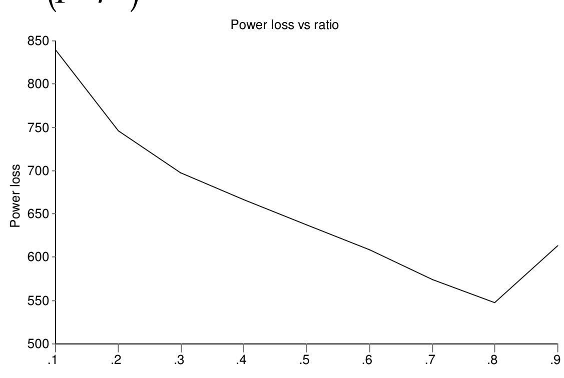
$$C_1 = 4.35 * 10^{11} \text{ N/(s.m}^2\text{)}$$

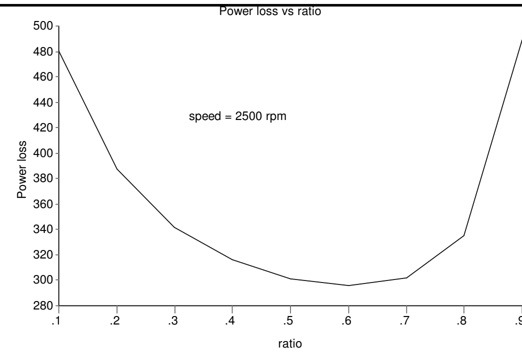
$$C_2 = 0.448 \text{ N.m}^2/s$$

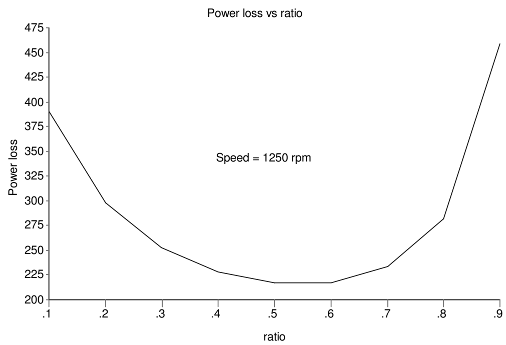


Example: W = 1000 N, ω=5000 rpm, R_0=100 mm, η=0.01 Pa.s, η_1=0.6, η_2=0.9, h_0=1mm. Optimize ratio (R_i/R_0) for minimum power loss

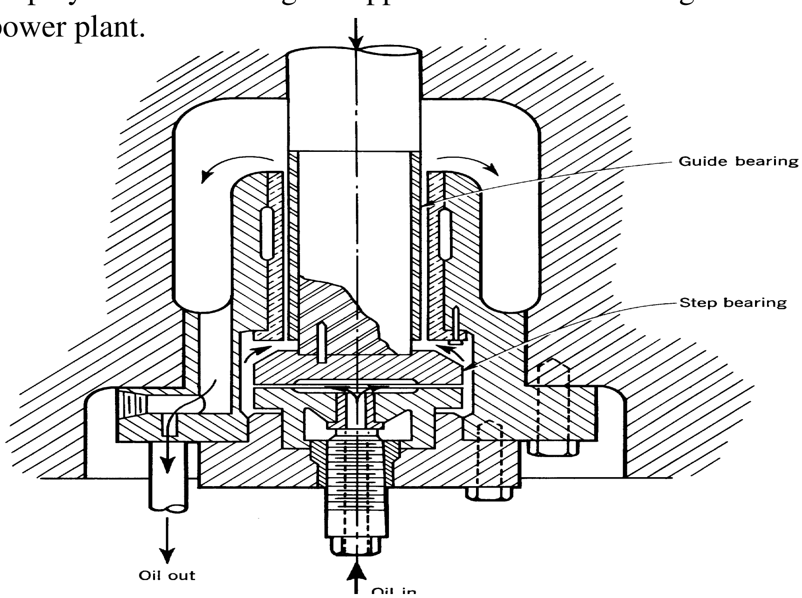
$$P_t = 353.5\frac{\log(1/r)}{\left(1-r^2\right)^2} + 478.7 * \left(1-r^4\right)$$







Step hydrostatic bearing to support vertical turbo-charger in power plant.



Restrictor

- In earlier slides, it was assumed that recess pressure was equal to supply pressure.

$$W = p_s.(\pi R_o^2)\left(\frac{1-R_i^2/R_o^2}{2.\log\left(R_o/R_i\right)}\right)$$

- This means change in load requires change in performance of pump.
- Pump performance can be regulated:
  - Manually
  - Automatically
- To automat the pump performance one needs sensor, amplifier, controller, etc.
- To reduce cost, often self regulating called restrictor is used.

Restrictor

- Constant flow restrictor $Q = \frac{\pi h_0^3 p_s}{6\eta}\frac{1}{\log(1/r_1)}$
  - If flow is constant, recess pressure and film thickness are related.
  - Increase in load, is balanced by increase in recess pressure and corresponding decrease in film thickness.
- Constant supply pressure restrictor
  - Recess pressure is kept lower than supply pressure
  - Drop in pressure, from supply pressure to recess pressure, depends is controlled by the fixed restrictor placed between supply manifold and the bearing.
  - Increase in load, reduces the flow by decreasing film thickness, recess pressure increases and equilibrium is restored.

Constant supply pressure restrictors

- Most commonly used restrictors are "capillary" and "orifice".
  - Capillary is relatively long and narrow opposed to and orifice which is short in the direction of flow.
  - In a capillary, flow occurs due to shearing and is dependent on viscosity of fluid, whereas flow in orifice is due to inertia and depends on density.
  - Flow in capillary is directly proportional to pressure difference and that in an orifice is dependent on square root of pressure difference.
  - Although the pumping power losses are higher for these types of compensation devices, the initial cost is much less.

$$Q_c = \frac{\Delta P \pi R_c^4}{8\eta l_c}$$

$$Q_O = \frac{\pi d_o^2}{4} C_D \sqrt{2.\Delta P / \rho}$$

A smaller flow produces a smaller pressure drop.

Hydrostatic Bearing Film Stiffness with Constant Feed Rate

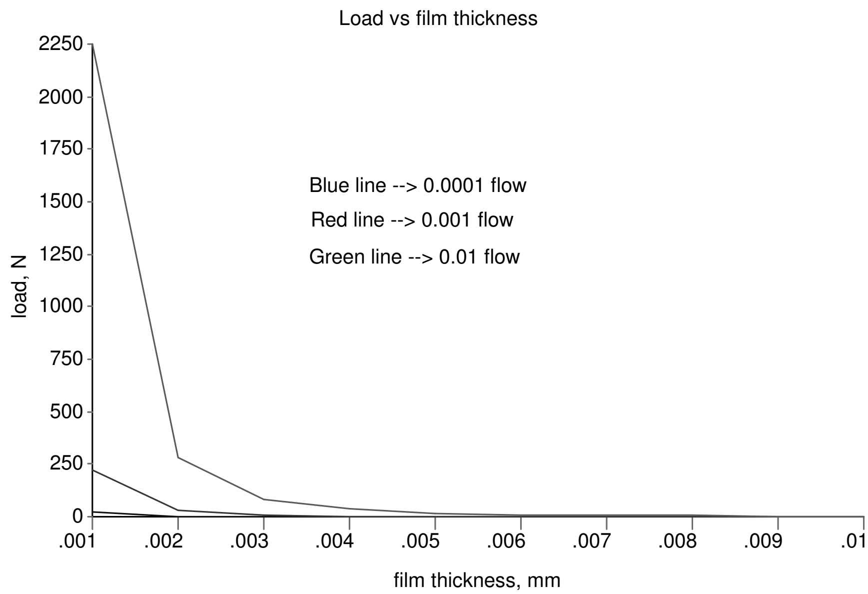
$$Q = \frac{\pi h_0^3 p_s}{6\eta} \frac{1}{\log(R_o / R_i)}$$

$$W = p_s.\left(\pi R_o^2\right)\left(\frac{1 - R_i^2 / R_o^2}{2.\log\left(R_o / R_i\right)}\right)$$

$$\Rightarrow W = 3\eta.R_o^2\left(1 - R_i^2 / R_o^2\right)\left(\frac{Q}{h_0^3}\right) \Rightarrow 3*0.01*(0.1)^2*(1-0.5^2)*\frac{Q}{h_o^3}$$

$$\text{Stiffness} K_1 = \frac{dW}{dh_0} = -\frac{3W}{h_0}$$

If W is doubled, and Q is kept constant, what will be relative change in film thickness?



Capillary Compensation

$$Q = \frac{\pi h_0^3 P_r}{6\eta}\frac{1}{\log(R_o / R_i)} \Rightarrow Q = B h_o^3 P_r \quad \text{-->}\ \mathrm{B} = 174$$

$$Q_c = \frac{(P_s - P_r)\pi R_c^4}{8\eta l_c} \Rightarrow Q_c = k_c (P_s - P_r) \text{---> assume}\ \eta = 0.01$$

$$\mathrm{r_c} = 0.15mm, l_c = 50 r_c$$

$$Q = Q_c$$

$$\Rightarrow \frac{P_r}{P_s} = \frac{k_c}{Bh_o^3 + k_c}$$

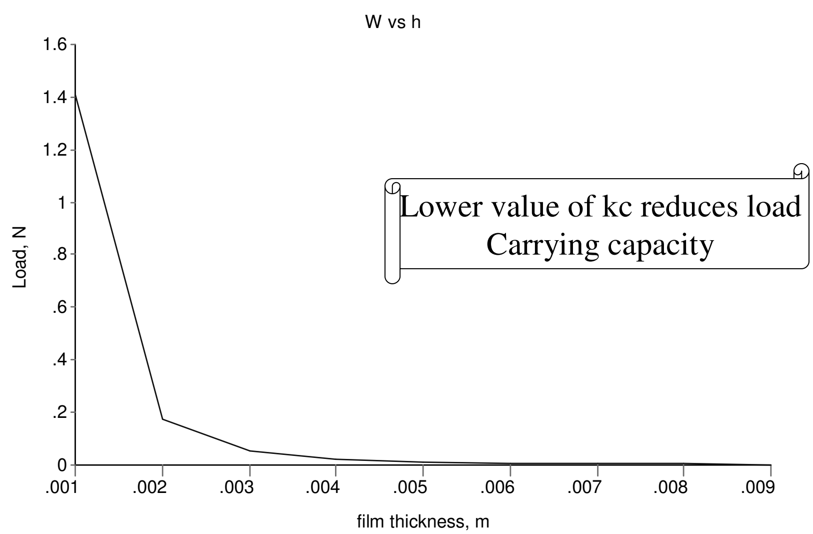
$$W = P_r.(\pi R_o^2)\left(\frac{1 - R_i^2 / R_o^2}{2.\log\left(R_o / R_i\right)}\right) \Rightarrow A_{eff} P_r$$

$$\Rightarrow W = A_{eff} P_s \frac{k_c}{Bh_o^3 + k_c} \qquad \mathrm{A_{eff}} = 0.039$$

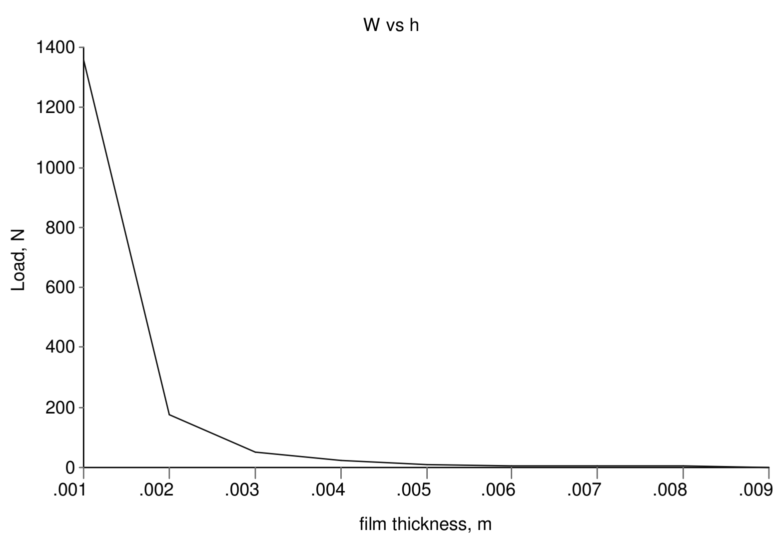
$$\frac{\partial W}{\partial h_o} = -A_{eff} P_s \frac{k_c}{Bh_o^3 + k_c}\frac{3Bh_o^2}{Bh_o^3 + k_c}$$

$$\text{or, } \frac{\partial W}{\partial h_o} = -\frac{3W}{h_o}\frac{Bh_o^3}{Bh_o^3 + k_c}$$

Lower value of kc increases stiffness



W vs h
Load, N
film thickness, m
Lower value of kc reduces load
Carrying capacity



W vs h
Load, N
film thickness, m

Orifice Compensation

$$Q_O = \frac{\pi d_o^2}{4} C_D \sqrt{2.(P_s - P_r)/\rho} \Rightarrow Q_O = k_o \sqrt{(P_s - P_r)}$$

$$Q = \frac{\pi h_0^3 P_r}{6\eta} \frac{1}{\log(R_o / R_i)} \Rightarrow Q = B h_o^3 P_r$$

$$Q = Q_o$$

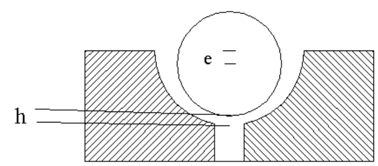
$$\Rightarrow \frac{P_r}{P_s} = \frac{k_o^2}{B^2 h_o^6 P_r + k_o^2}$$

$$W = P_r.(\pi R_o^2)\left(\frac{1 - R_i^2 / R_o^2}{2.\log\left(R_o / R_i \right)}\right) \Rightarrow A_{eff} P_r$$

$$\Rightarrow W = A_{eff}\left[\frac{-k_o^2 + k_o\sqrt{k_o^2 + 4B^2 h_o^6 P_s}}{2B^2 h_o^6}\right]$$

Hydrostatic Lift

- Useful to avoid metal to metal contact under heavy static load conditions. Ex. Synchronous condenser, rolling mills, etc.
- How to estimate load capacity ?



Elemental flow rate

$$\delta q = -\frac{h^3}{12\eta}\frac{dp}{rd\theta}.b$$

$$\mathrm{h} = \mathrm{C_r} - e\cos\theta$$

- Trial and error method
  - Good for first of its kind.
- Numerical modeling and simulation
  - Assume a shaft of radius r being floated in a bearing of radius R by oil pumped through a slot at pressure P_S

Hydrostatic lift………………..

$$h = C_r - e\cos\theta \rightarrow h = C_r(1-\varepsilon\cos\theta)$$

Elementalflow rate

$$\varepsilon = e / C_r$$

$$\delta q = -\frac{C_r^{\,3}.(1-\varepsilon\cos\theta)^3}{12\eta}\frac{dp}{rd\theta}.b$$

Elemental pressure rise

$$dp = -\frac{12r\eta q_1}{bC_r^3}\frac{d\theta}{(1-\varepsilon\cos\theta)^3}$$

Generalsolution

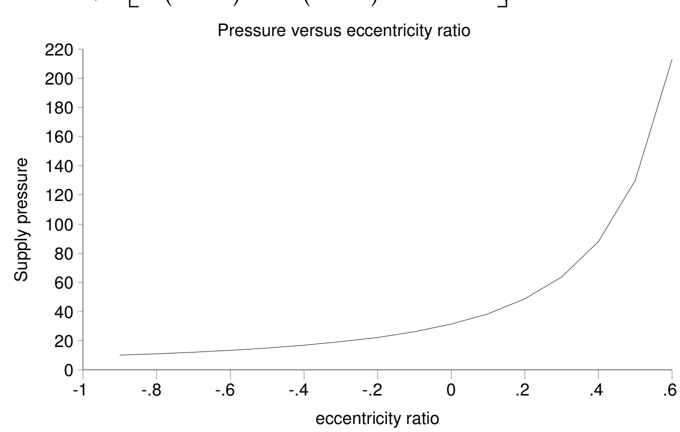
$$P = -\frac{12r\eta q_1}{bC_r^3}\left[\frac{\varepsilon.\sin\theta.(4-\varepsilon^2-3\varepsilon\cos\theta)}{2.(1-\varepsilon^2)^2.(1-\varepsilon\cos\theta)^2}+\frac{2+\varepsilon^2}{2.(1-\varepsilon^2)^{2.5}}\cos^{-1}\left(\frac{\varepsilon-\cos\theta}{1-\varepsilon\cos\theta}\right)+D\right]$$

constant of integration, D, can be evaluated using $P = 0$ at $\theta = 90^0$

$$D = -\left[\frac{\varepsilon.(4-\varepsilon^2)}{2.(1-\varepsilon^2)^2}+\frac{2+\varepsilon^2}{2.(1-\varepsilon^2)^{2.5}}\cos^{-1}(\varepsilon)\right]$$

supply pressure P_s can be determined using $P = P_s$ at $\theta = 0^0$

$$P_s = \frac{12r\eta q_1}{bC_r^3}\left[\frac{\varepsilon.(4-\varepsilon^2)}{2.(1-\varepsilon^2)^2}+\frac{2+\varepsilon^2}{2.(1-\varepsilon^2)^{2.5}}\cos^{-1}(\varepsilon)\right]$$



Negative value of eccentricity ratio, describe the journal position when it is above the bearing center.

Load Carrying Capacity

Pressure p acts on area $rd\theta.b$ and vertical component of force $prd\theta.b.\cos\theta$ will balance the applied load W

$$\therefore \mathrm{W} = 2\int_0^{\pi/2} \mathrm{br\ p}\cos\theta.\mathrm{d}\theta$$

$$W = -\frac{24r^2\eta q_1}{C_r^3}\int_0^{\pi/2}\left[\frac{\varepsilon.\sin\theta.\left(4-\varepsilon^2-3\varepsilon\cos\theta\right)}{2.\left(1-\varepsilon^2\right)^2.\left(1-\varepsilon\cos\theta\right)^2}+\frac{2+\varepsilon^2}{2.\left(1-\varepsilon^2\right)^{2.5}}\cos^{-1}\left(\frac{\varepsilon-\cos\theta}{1-\varepsilon\cos\theta}\right)\right]\cos\theta.d\theta$$

$$\rightarrow W = \frac{12\eta r^2 q_1}{C_r^3}\left[\frac{2-\varepsilon}{\left(1-\varepsilon\right)^2}\right]$$